1. (10 points) Suppose a group of researchers conducts an experiment involving $n = 25$ observations. The sample mean of these 25 observations is 85.5 and the sample standard deviation is 46.25.

(a) Form a 95% confidence interval for the population mean.

(b) Explain what this interval represents, and in particular to what event does the 95% refer?

(c) (True or False) The chance that the population mean is contained in $(67,104)$ is 95%?
2. (20 points) In 1909, the Brooklyn Standard Union published a list of people eligible for the position of stenographer and typist. Each eligible typist was given a typing test and ranked according to their accuracy. The test consisted of a 250 word passage from a business letter. Each person was asked to reproduce the same text, and their score was simply the proportion of words (out of 250) that they typed correctly. Typists were then divided into “grades” based on these scores. Grade 1 was the best, requiring that a person pass the test with a score of 95% or better. Below we present the distribution of scores among those typists in Grade 1; that is, the probability that a randomly selected typist from Grade 1 will have a score of $x$ for $x = 95, 96, \ldots, 100$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>95</th>
<th>96</th>
<th>97</th>
<th>98</th>
<th>99</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>0.35</td>
<td>0.32</td>
<td>0.16</td>
<td>0.11</td>
<td>0.05</td>
<td>0.01</td>
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</table>

(a) Let $X$ denote the score of a randomly selected typist from Grade 1. Compute the expected value and the variance of $X$.

Each time a typist takes the test, they are likely to do a little better or a little worse. Suppose that the number of correctly typed words on a typing test is well-described by a binomial distribution; $p$ represents the chance that a word is typed correctly, and $n$ is the number of words on the test.

Consider two of the less-skilled typists from the 1909 list. Suppose that for E. Gustavson the probability of correctly typing a word is $p_1 = 0.8$; and suppose that for G. Brody the probability of correctly typing a word is $p_2 = 0.7$.

Given a typing test consisting of $n = 100$ words, let $\hat{P}_1$ denote the proportion of words (out of $n$) that E. Gustavson types correctly; and let $\hat{P}_2$ denote the proportion of words (out of $n$) that G. Brody types correctly.

(b) We will assume that $\hat{P}_1$ and $\hat{P}_2$ are independent and have approximately normal distributions. Why is this reasonable given our assumption about the number of correctly-typed words on a typing test?
(c) What are the means and variances of $\hat{P}_1$ and $\hat{P}_2$?

(d) Using the normal approximation above, compute the probability that E. Gustavson scores lower than G. Brody on the typing test. That is, compute $\Pr(\hat{P}_1 < \hat{P}_2)$.

(e) How long should a typing test be so that the chance that E. Gustavson scores lower than G. Brody is 0.01? That is, find $n$ so that $\Pr(\hat{P}_1 < \hat{P}_2) = 0.01$. (The probability table we provided will give you 3 possible values, choose the smallest.)

3. (20 points) A manufacturer of polyurethane sponges is interested in starting a program to monitor the quality of its product. An automated visual inspection system is developed that collects digital images of the surface of the sponges and estimates the diameters of the pores (the holes). Initially, the engineers studying the problem decide that to have good absorption properties the diameters of the pores should be smaller than 4 mm.

For the manufacturer’s fabrication process, the engineers believe that the diameter of a randomly selected pore can be expressed as a random variable $D = |X|$, where $X$ has a normal distribution with mean 0 and standard deviation 2.5 mm.
(a) Express the engineers’ condition on pore diameter in terms of an event involving a standard normal random variable. (Keep in mind that $D$ is written in terms of the absolute value of $X$.)

(b) Compute the probability that a randomly selected pore on a sponge will satisfy the engineers’ condition on pore diameter.

(c) The engineers propose a quality test in which the visual inspection system randomly selects 20 pores on a sponge and measures their diameters. The number of pores in the sample that have diameters less than 4 mm has what kind of distribution? What are its parameters?
(d) The engineers decide that a sponge will absorb properly if at least 90% of its pores have diameters less than 4mm. What is the chance that 90% or more of the pores in a random sample of 20 will have diameters smaller than 4mm? (Use the distribution you identified in the previous problem and compute the exact probability.)

(e) After several weeks of inspecting sponges, the engineers realize that very small pores can also reduce the absorption capabilities of a sponge. In addition to wanting pores with diameters smaller than 4mm, they want the pores to have diameters larger than 0.5 mm. Express the engineers’ new condition that \( 0.5 < D < 4 \) in terms of an event involving a standard normal random variable.

(f) Compute the probability that a randomly selected pore on a sponge will satisfy the engineers’ new condition on pore diameter.