Introduction to Statistical Reasoning

Instructor: Ivo Dinov, Asst. Prof. in Statistics and Neurology

University of California, Los Angeles, Spring 2002
http://www.stat.ucla.edu/~dinov/

Part IV: Chances, Variability, Probabilities and Proportions

- Chances and chance variability
- Where do probabilities come from?
- Simple probability models
- Probability rules
- Conditional probability
- Statistical independence

Let's Make a Deal Paradox – aka, Monty Hall 3-door problem

This paradox is related to a popular television show in the 1970's. In the show, a contestant was given a choice of three doors/cards of which one contained a prize (diamond). The other two doors contained gag gifts like a chicken or a donkey (clubs).

Let's Make a Deal Paradox.

The intuition of most people tells them that each of the doors, the chosen door and the unchosen door, are equally likely to contain the prize so that there is a 50-50 chance of winning with either selection? This, however, is not the case.

The probability of winning by using the switching technique is 2/3, while the odds of winning by not switching is 1/3. The easiest way to explain this is as follows:

Let's Make a Deal Paradox.

The probability of picking the wrong door in the initial stage of the game is 2/3.

If the contestant picks the wrong door initially, the host must reveal the remaining empty door in the second stage of the game. Thus, if the contestant switches after picking the wrong door initially, the contestant will win the prize.

The probability of winning by switching then reduces to the probability of picking the wrong door in the initial stage which is clearly 2/3.

StatGames.exe (Make a Deal Paradox)
**Chance**

- The **chance** of something happening gives the percentage of time it is expected to happen, when the basic process is repeatedly performed.
- E.g., What is the chance of getting an ace (1) if we roll a regular 6-face (hexagonal) die?
- Chances are always between 0% - 100%.
- The chance of an event is equal to 100% - the chance of the opposite (complementary) event.
- E.g., Chance(getting 1) = 100 – Chance(2 or 3 or 4 or 5 or 6 turns up).

**Coin toss experiments (Head vs. Tail)**

- The **law of averages** about the behavior of coin tosses – the relative proportion (relative frequency) of heads-to-tails in a coin toss experiment becomes more and more stable as the number of tosses increases. The law of averages applies to relative frequencies not absolute counts of #H and #T.
- Two widely held misconceptions about what the **law of averages** about coin tosses:
  - Differences between the actual numbers of heads & tails becomes more and more variable with increase of the number of tosses – a seq. of 10 heads doesn’t increase the chance of a tail on the next trial.
  - Coin toss results are fair, but behavior is still unpredictable.

**Coin Toss Models**

- Is the coin tossing model adequate for describing the **sex order** of children in families?
  - This is a **rough model** which is not exact. In most countries rates of B/G is different; from 48% … to 52%, usually. Birth rates of boys in some places are higher than girls, however, female population seems to be about 51%.
  - **Independence**, if a second child is born the chance it has the same gender (as the first child) is slightly bigger.

**Two die throw example**

- What is the chance that the sum of the numbers, turning up when 2 dice are rolled, is equal to 8?
- Do the HTML Java-applet: UCLA_ChanceApplet/DiceApplet.htm

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**Data from a “random” draw**

366 cylinders (for each day in the year) for the US Vietnam war draft. The N-th drawn number, corr. to one B-day, indicating order of drafting.

So, people born later in the year tend to have lower lottery numbers and a bigger chance of actually being drafted.
Types of Probability

- Probability models have two essential components (sample space, the space of all possible outcomes from an experiment; and a list of probabilities for each event in the sample space). Where do the outcomes and the probabilities come from?
- Probabilities from models – say mathematical/physical description of the sample space and the chance of each event. Construct a fair die tossing game.
- Probabilities from data – data observations determine our probability distribution. Say we toss a coin 100 times and the observed Head/Tail counts are used as probabilities.
- Subjective Probabilities – combining data and psychological factors to design a reasonable probability table (e.g., gambling, stock market).

CA State Lottery – Supper Lotto Plus

- California Lotto, chose 5 out of 47 and choose one Mega from [1 : 27], fee $1, your odds are 1 in 41,416,353!
- Why?
- \(47\text{-choose-5} = \frac{47!}{(47-5)!5!}\) ➔
- \(47\text{-choose-5} \times 27 = 1,533,939 \times 27 = 41,416,353\)

Sample spaces and events

- A **sample space**, \(S\), for a random experiment is the set of all possible outcomes of the experiment.
- Examples?
- An **event** is a collection of outcomes.
- Examples?
- An event **occurs** if any outcome making up that event occurs.
- Examples?

The complement of an event

- The **complement** of an event \(A\), denoted \(\bar{A}\), occurs if and only if \(A\) does not occur.

Combining events – all statisticians agree on

- “**A or B**” contains all outcomes in \(A\) or \(B\) (or both).
- “**A and B**” contains all outcomes which are in both \(A\) and \(B\).

Probability distributions

- Probabilities always lie between 0 and 1 and they sum up to 1 (across all simple events).
- \(pr(A)\) can be obtained by adding up the probabilities of all the outcomes in \(A\).
- \[ pr(A) = \sum_{E\text{ outcome in event } A} pr(E) \]
Job losses in the US in $1,000, 1987-1991

<table>
<thead>
<tr>
<th>Workplace Position</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>moved/closed</td>
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</tr>
<tr>
<td>Slack work</td>
<td></td>
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<tr>
<td>abolished</td>
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<table>
<thead>
<tr>
<th>Reason for Job Loss</th>
<th>Workplace moved/closed</th>
<th>Slack work</th>
<th>Position abolished</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>1,703</td>
<td>1,196</td>
<td>548</td>
<td>3,447</td>
</tr>
<tr>
<td>Female</td>
<td>1,210</td>
<td>564</td>
<td>363</td>
<td>2,137</td>
</tr>
<tr>
<td>Total</td>
<td>2,913</td>
<td>1,760</td>
<td>911</td>
<td>5,584</td>
</tr>
</tbody>
</table>

Reason for Job Loss

- Moved/closed
- Slack work abolished

Job losses raw-data vs. proportions

<table>
<thead>
<tr>
<th>Workplace Position</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
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<tr>
<td>Slack work</td>
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<table>
<thead>
<tr>
<th>Reason for Job Loss</th>
<th>Workplace moved/closed</th>
<th>Slack work</th>
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<th>Row totals</th>
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<td>Male</td>
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</tr>
</tbody>
</table>

What is a sample space? What are the two essential criteria that must be satisfied by a possible sample space? (completeness — every outcome is represented; and uniqueness — no outcome is represented more than once.)

What is an event? (collection of outcomes)

- If A is an event, what do we mean by its complement, $\bar{A}$? When does $\bar{A}$ occur?
- If A and B are events, when does A or B occur? When does A and B occur?

Example of probability distributions

- Tossing a coin twice. Sample space $S=\{HH, HT, TH, TT\}$, for a fair coin each outcome is equally likely, so the probabilities of the 4 possible outcomes should be identical, $p$. Since, $p(HH)=p(HT)=p(TH)=p(TT)=p$ and $p = \frac{1}{4} = 0.25$.

Proportion vs. Probability

- How do the concepts of a proportion and a probability differ? A proportion is a partial description of a real population. The probabilities give us the chance of something happening in a random experiment. Sometimes, proportions are identical to probabilities (e.g., in a real population under the experiment choose-a-unit-at-random).

- See the two-way table of counts (contingency table) E.g., choose-a-person-at-random from the ones laid off, and compute the chance that the person would be a male, laid off due to position-closing. We can apply the same rules for manipulating probabilities to proportions, in the case where these two are identical.

Rules for manipulating Probabilities

For mutually exclusive events, $pr(A \text{ or } B) = pr(A) + pr(B)$

$pr(A \text{ or } B) = pr(A)+pr(B) - pr(A \& B)$
Betting on the event $E = \{\text{In 4 die rolls at least 1 ace turns up} \}$, $B = \{\text{In 24 rolls of a pair of dice, at least one double-ace shows up} \}$.

**Claim**: $P(E) = P(B)$?!

**Reasoning**:
- $E$: 1 roll gives a chance $1/6$ for an ace! So, in 4 rolls we have $4 \times 1/6 = 2/3$ to get at least 1 ace!
- $B$: In one roll of a pair of dice, chance of a double-ace is $1/36$. So in 24 rolls we have $24 \times 1/36 = 2/3$ chance.

**Experience** showed $P(E) > P(B)$!!!

What’s wrong? Well, extrapolating these arguments we get that the chance of getting 1 ace in 6 rolls is $6 \times 1/6 = 1$? Obviously, incorrect!

The chance of winning (getting at least one ace) is hard to compute, but can we calculate the chance of loosing – the complement event?!!

Then $\text{chance-of-winning} = 1 - \text{chance-of-loosing}$.

$E^c$, complement of $E$, = {none of 4 rolls shows an ace}.

In one roll, chance of loosing is $5/6$, no ace turns up.

2 die rolls are independent, hence we can use the multiplication rule, Chance of no ace in two rolls is $(5/6)^2$. Similarly, chance of 4 rolls with no ace, the probability $P(E^c) = (5/6)^4 \approx 0.482$.

Game 2: Pair-of-dice: Chance of no-ace in 1 roll is $35/36$. Hence, $P(\text{no-Ace in 24 rolls}) = (35/36)^{24} \approx 0.509$.

$P(\text{at-least-1-ace-in-4-rolls}) = 1 - 0.482 = 0.518 >> P(\text{at-least-1-double-ace-in-24-rolls}) = 1 - 0.509 = 0.491$. 

The conditional probability of $A$ occurring given that $B$ occurs is given by

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

**Multiplication rule**

- $P(A$ and $B) = P(A \mid B)P(B) = P(B \mid A)P(A)$

- Basic Rules for computing Probabilities

$$P(A) = 1 - P(\overline{A})$$

Properties of probabilities.

$\{p_i\}_{i=1}^n$ define probabilities $\iff p_i \geq 0; \sum p_i = 1$
A tree diagram for computing conditional probabilities

\[ P(A \cap B) = P(A \mid B) \times P(B) = P(B \mid A) \times P(A) \]

Suppose we draw 2 balls at random one at a time without replacement from an urn containing 4 black and 3 white balls, otherwise identical. What is the probability that the second ball is black? Sample Spc?

\[ P(\{2-nd \text{ ball is black}\}) = P(\{2-nd \text{ black} \} \cap \{1-st \text{ is black}\}) + P(\{2-nd \text{ is black} \} \cap \{1-st \text{ is white}\}) = \frac{3}{6} \times \frac{4}{5} + \frac{4}{6} \times \frac{3}{5} = \frac{4}{7}. \]

A tree diagram for computing conditional probabilities

Conditional probabilities and 2-way tables

- Many problems involving conditional probabilities can be solved by constructing two-way tables
- This includes reversing the order of conditioning

\[ P(A \cap B) = P(A \mid B) \times P(B) = P(B \mid A) \times P(A) \]

HIV – reconstructing the contingency table

\[
\begin{array}{c|cc}
\text{HIV and Positive} & \text{HIV} & \text{Not HIV} \\
\hline
\text{Positive} & 98 \times 0.01 & 93 \times 0.99 \\
\hline
\text{Negative} & 1 \times 0.01 & 1 \times 0.99 \\
\hline
\text{Total} & 99 & 93 \\
\end{array}
\]

\[ \text{Test result} \]

\[ \begin{array}{c|cc}
\text{HIV} & \text{Positive} & \text{Negative} \\
\hline
\text{Positive} & 98 \times 0.01 & 93 \times 0.99 \\
\hline
\text{Negative} & 1 \times 0.01 & 1 \times 0.99 \\
\hline
\text{Total} & 99 & 93 \\
\end{array} \]

\[ \text{Test Result} \]

\[ \begin{array}{c|cc|c}
\text{Disease status} & \text{Positive} & \text{Negative} & \text{Total} \\
\hline
\text{HIV} & 98 & 0.01 & 99 \\
\text{Not HIV} & 99 & 0.99 & 99 \\
\hline
\text{Total} & 197 & 1.00 & 197 \\
\end{array} \]

\[ \text{HIV} \quad \text{cont.} \]

Number of Individuals

<table>
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<th>HIV patients</th>
<th>Total</th>
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<td>0</td>
<td>202</td>
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<tr>
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<td>73</td>
<td>2</td>
<td>75</td>
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<td>3 - 3.99</td>
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<td>5 - 5.99</td>
<td>2</td>
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<td>17</td>
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<tr>
<td>6 - 6.99</td>
<td>2</td>
<td>36</td>
<td>38</td>
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<tr>
<td>12+</td>
<td>0</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>Total</td>
<td>297</td>
<td>88</td>
<td>385</td>
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</table>
**Examples**

Let \( X \) be the sum of spots from rolling 4 fair dice. Determine the expected value, the variance, and the standard deviation of the random variable \( X \).

\[
E(Y) = \sum_{y} y \times P(Y = y) \quad ; \quad Var(Y) = \frac{1}{N-1} \sum_{y} (y - E(Y))^2
\]

- Because the rolls are independent:
  - \( Var(X) = Var(X1) + Var(X2) + Var(X3) + Var(X4) \)
  - The variance for any single roll is: \( (1/6)^2(1-3.5)^2 + (1/6)^2(2-3.5)^2 + (1/6)^2(3-3.5)^2 + (1/6)^2(4-3.5)^2 + (1/6)^2(5-3.5)^2 + (1/6)^2(6-3.5)^2 = 3.5 \)
  - So, \( Var(X) = 4 \times 3.5 = 14 \). SD(X) = sqrt(Var(X)) = 3.74.
  - So, from 4 dice, the expected value (Sum) is 14, with a SE of 3.74

**Statistical independence**

- Events \( A \) and \( B \) are **statistically independent** if knowing whether \( B \) has occurred gives no new information about the chances of \( A \) occurring,
  - i.e. \( \Pr(A | B) = \Pr(A) \)
- Similarly, \( \Pr(B | A) = \Pr(B) \), since
  - \( \Pr(B) = \Pr(B \& A) / \Pr(A) = \Pr(A \& B) / \Pr(A) = \Pr(B) \)
- If \( A \) and \( B \) are **statistically independent**, then
  - \( \Pr(A \& B) = \Pr(A) \times \Pr(B) \)

**Examples – Birthday Paradox**

- Let \( X \) be the sum of spots from rolling 4 fair dice. Determine the expected value, the variance, and the standard deviation of the random variable \( X \).
- \( E(Y) = \sum_{y} y \times P(Y = y) = \text{Sum Of All Obs'}/\text{Total Number} \)
- For each die,
  - \( E(X1) = 1 \times 1/6 + 2 \times 1/6 + 3 \times 1/6 + 4 \times 1/6 + 5 \times 1/6 + 6 \times 1/6 = 3.5 \)
  - \( E(X) = E(X1) + \ldots + E(X4) = 3.5 + 3.5 + 3.5 + 3.5 = 14 \)
- Let \( X = X1 + X2 + X3 + X4 \), where \( X1, X2, X3, X4 \) be the random variables for the spots showing on the 1st, 2nd, 3rd, and 4th dice, respectively. [ \( E(X) = \text{Sum(X values times P(x))} \)]

**Examples**

- Two coins are given. One is fair (P(H)=0.5) and the other is biased (P(H)=2/3). One of the coins is tossed once, resulting in H. The other is tossed three times, resulting in two heads. Which coin is more likely to be the biased one?
- We won’t look for the probability of the first or the second coin being the biased one, rather we look for the probability of the given outcomes in two different cases: the first coin being the fair one, and the second—the biased one, and vice versa.
- If we assume that the first coin is fair, then the probability of the heads is 1/2. The second coin must be the biased one, and the probability of it coming up with 2 heads and 1 tail in three tosses is \( 3/2 \times 3/2 \times 3/2 \times 1/2 = 4/9 \). Note that there are three ways to get 2 heads: HHT, HTH, THH, the probability of each being 1/27. Thus, the probability of both coins coming up with the given results is 2/9.
- If, on the other hand, the first coin is the biased one, and the second coin is fair the probability of them resulting in the combination given in the problem is \( 2/3 \times 1/2 \times 1/2 \times 1/2 = 1/4 \), or 2/8 > 2/9. Therefore, it is more probable that the first coin is the biased one.

**Examples – People vs. Collins**

- The first occasion where a conviction was made in an American court of law, largely on statistical evidence, 1964. A woman was mugged and the offender was described as a wearing dark clothes, with blond hair in a pony tail who got into a yellow car driven by a black male accomplice with mustache and beard. The suspect brought to trial were picked out in a line-up and fit all of the descriptions. Using the *product rule for probabilities* an expert witness computed the chance that a random couple meets these characteristics, as 1:12,000,000.

- **Frequencies assumed by the prosecution:**
  - Yellow car
  - Man with mustache
  - Black man with beard
  - Girl with blond hair
  - Girl with ponytail
  - Interracial couple in car
  - 1/9
  - 1/4
  - 1/10
  - 1/10
  - 1/1000

- **The Birthday Paradox:** In a random group of \( N \) people, what is the change that at least two people have the same birthday?
- E.x., if \( N=23 \), P>0.5. Main confusion arises from the fact that in real life we rarely meet people having the same birthday as us, and we meet more than 23 people.
- The reason for such high probability is that any of the 23 people we meet more than 23 people.
- Assume there are 365 days in a year, \( P(\text{one-particular-pair-same-birthday}) = 1/365 \).
- The number of birthday combinations is \( (N \begin{pmatrix} 2 \end{pmatrix}) \). The chance of each combination is 1/365. The chance of at least two people having the same birthday is the sum of the chance of each combination.
- \( P(\text{at-least-one-success}) = 1-0.59 \quad \text{or 0.41} \) high.
- Note: for \( N=42 \), P=0.9 ...
Summary

- What does it mean for two events $A$ and $B$ to be statistically independent?
- Why is the working rule under independence, $P(A \text{ and } B) = P(A) \cdot P(B)$, just a special case of the multiplication rule $P(A \& B) = P(A | B) \cdot P(B)$?
- Mutual independence of events $A_1, A_2, A_3, \ldots, A_n$ if and only if $P(A_1 \& A_2 \& \ldots \& A_n) = P(A_1)P(A_2)\ldots P(A_n)$

Binomial Distribution

- The distribution of the number of heads in $n$ tosses of a biased coin is called the Binomial distribution.

The two-color urn model

- $N$ balls in an urn, of which there are $M$ black balls
- $N - M$ white balls
- Sample $n$ balls and count $X = \#$ black balls in sample

We will compute the probability distribution of the R.V. $X$

The biased-coin tossing model

- Perform $n$ tosses and count $X = \#$ heads
- We also want to compute the probability distribution of this R.V. $X$!
- Are the two-color urn and the biased-coin models related? How do we present the models in mathematical terms?

The answer is: Binomial distribution

- The distribution of the number of heads in $n$ tosses of a biased coin is called the Binomial distribution.

| Binomial($N$, $p$) – the probability distribution of the number of Heads in an $N$-toss coin experiment, where the probability for Head occurring in each trial is $p$. E.g., Binomial(6, 0.7) |
|---|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Individual $\text{pr}(X=x)$ | 0.001 | 0.010 | 0.069 | 0.185 | 0.324 | 0.303 | 0.118 |
| Cumulative $\text{pr}(X\leq x)$ | 0.001 | 0.011 | 0.070 | 0.256 | 0.580 | 0.882 | 1.000 |

For example $P(X=0) = P(\text{all 6 tosses are Tails}) = (1 - 0.7)^6 = 0.3^6 = 0.001$
Binary random process

The biased-coin tossing model is a physical model for situations which can be characterized as a series of trials where:

- each trial has only two outcomes: success or failure;
- \( p = P(\text{success}) \) is the same for every trial; and
- trials are independent.

- The distribution of \( X = \) number of successes (heads) in \( N \) such trials is

\[
\text{Binomial}(N, p)
\]

Sampling from a finite population – Binomial Approximation

If we take a sample of size \( n \)

- from a much larger population (of size \( N \))
- in which a proportion \( p \) have a characteristic of interest, then the distribution of \( X \), the number in the sample with that characteristic,

- is approximately \( \text{Binomial}(n, p) \).

(Operating Rule: Approximation is adequate if \( n / N < 0.1 \).)

- Example, polling the US population to see what proportion is/has been married.

Review

- For what types of situation is the urn-sampling model useful? For modeling binary random processes. When sampling with replacement, Binomial distribution is exact, whereas in sampling without replacement Binomial distribution is an approximation.

- For what types of situation is the biased-coin sampling model useful? Defective parts. Approval poll of cloning for medicinal purposes. Number of boys in 151 presidential children (90).

- Give the three essential conditions for its applicability. (two outcomes; same \( p \) for every trial; independence)

The Expected value

- The expected value:

\[
E(X) = \sum x \cdot P(x)
\]

\[
e\text{all } x
\]

- = Sum of \( (\text{value times probability of value})\)

- = Sum of all Obs’s / Total Number

Example

A couple wants to have children, but they insist on stopping when they have at least one of each gender or at most 3 children example, where \( X = \) (number of Girls) we have:

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x \cdot P(x) )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{5}{8} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
</tbody>
</table>

\[
E(X) = \sum x \cdot P(x)
\]

\[
= 0 \cdot \frac{1}{8} + 1 \cdot \frac{5}{8} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8}
\]

\[
= 1.25
\]

The expected value as the point of balance

\[
\mu_X
\]

The mean \( \mu_X \) is the balance point.
Examples

- 35% of all its 475 pilots are over 40. The company is to select a random sample of 25 pilots. \( X = \# \) pilots over 40 years in sample.
- State the value of the parameter(s) of this distribution.
  - Binomial(25, 0.35)
- Assuming that the Binomial distribution you have described above is an appropriate model for \( X \), find the probability that:
  1. more than 7 of the pilots selected are over 40 years of age.
  2. 5 or 6 of the pilots selected are over 40 years of age.
  3. between 13 and 18 (inclusive) of the pilots selected are over 40 years of age.

\[
\begin{align*}
\text{P}(X=5 \text{ or } X=6) &= \binom{25}{5} 0.35^5 \times 0.65^{20} + \binom{25}{6} 0.35^6 \times 0.65^{19} \\
&= 0.0514 + 0.091 = 0.142
\end{align*}
\]

\[
\text{P}(13 \leq X \leq 18) = 0.06, \text{ From the online table.}
\]

Examples

- 35% of all its 475 pilots are over 40 years of age. The company is to select a random sample of 25 pilots. \( X = \# \) pilots over 40 years in this sample.
- How many of the pilots selected would you expect to be over 40 years of age? What is the standard deviation of \( X \)?
  - \( E(X) = n \times p = 25 \times 0.35 = 8.75 \)
  - \( SD(X) = \text{Var}(X) = n \times p \times (1-p) = 5.6875 \)
  - \( SD(X) = 2.385 \)
- Why would the airline company be interested in the variable “pilots over 40 years of age”? Suggest another variable the company may be interested in measuring? Briefly justify your answer. (GO TO Ch. 21, CI’s)