Two supplements:
(1) Population version of ANOVA.
For completeness, we derive the ANOVA identity here for population version:

\[ \text{cov}(\mathbf{x}) = \text{cov}(E(\mathbf{x}|Y)) + E(\text{cov}(\mathbf{x}|Y)) \]

Here \( \mathbf{x} \) is a \( p \)-dimensional random vector and \( Y \) is a random variable (or vector). First write

\[ \mathbf{x} = E(\mathbf{x}|Y) + (\mathbf{x} - E(\mathbf{x}|Y)) \]

Because \( E[\mathbf{x} - E(\mathbf{x}|Y)] = 0 \), it follows that \( \text{cov}(E(\mathbf{x}|Y), \mathbf{x} - E(\mathbf{x}|Y)) = E[E(\mathbf{x}|Y)(\mathbf{x} - E(\mathbf{x}|Y))'|Y] = E(E(\mathbf{x}|Y)[E((\mathbf{x} - E(\mathbf{x}|Y))|Y]) = E[(E(\mathbf{x}|Y)0] = 0. \]

Thus we have \( \text{cov}(\mathbf{x}) = \text{cov}(E(\mathbf{x}|Y)) + \text{cov}(\mathbf{x} - E(\mathbf{x}|Y)) \) Now

\[ \text{cov}(\mathbf{x} - E(\mathbf{x}|Y)) = E(\mathbf{x} - E(\mathbf{x}|Y))(\mathbf{x} - E(\mathbf{x}|Y))' \]

\[ = E(E[(\mathbf{x} - E(\mathbf{x}|Y))(\mathbf{x} - E(\mathbf{x}|Y))'|Y]) = E(\text{cov}(\mathbf{x}|Y)) \]

We have derived the ANOVA identity.
(2) Finding $E(x|\beta'x)$, under $x = 0$ and the conditional linearity assumption that for any vector $b$, there exists a constant $c$ such that

$$E(b'x|\beta'x) = c\beta'x$$

(note $c$ may depend on $b$).

First suppose $cov(x) = I$. Then for any vector $b$, $cov(b'x, \beta'x) = b'cov(x)\beta = b'\beta$.

Now, for any two random variables with mean zero, say $V, W$, it is clear that $cov(W, V) = E(VW) = E(E(VW|V)) = E(VE(W|V))$. Taking $V = \beta'x$ and $W = b'x$, we see that $cov(b'x, \beta'x) = E(\beta'x E(b'x|\beta'x))$, which, due to the linearity condition, equals to $E(\beta'x c \beta'x) = c\beta'\beta$.

Therefore, we have shown that $c = b'\beta(\beta'\beta)^{-1}$ Since this is true for any $b$, it must hold that $E(x|\beta'x) = \beta(\beta'\beta)^{-1}\beta'x$ We have seen that $E(x|\beta'x)$ must be proportional to $\beta$.

For the general covariance matrix of $x$, we can obtain $b'\Sigma_x \beta = c\beta'\Sigma_x \beta$. Thus $E(x|Y) = \Sigma_x \beta(\beta'\Sigma_x \beta)^{-1}\beta'x$, which is proportional to $\Sigma_x \beta$. 
