Statistics 10  
Instructor: Nicolas Christou  

Test for the difference between two population means

Example 1:
An investigator wishes to test whether there is a difference of 10 mg/dl in cholesterol level between a control group and a diet intervention group (low-fat). The sample means of the cholesterol level for $n = 100$ subjects in each group are $\bar{x}_1 = 230$ mg/dl and $\bar{x}_2 = 215$ mg/dl. Assume that for both groups the standard deviation is $\sigma = 16$ mg/dl. Test the hypothesis that the difference is more than 10 mg/dl.

Example 2:
A study was conducted to investigate the effect of an oral antiplaque rinse on plaque buildup on teeth. For this study, 14 subjects were divided into 2 groups of 7 subjects each. Both groups were assigned to use oral rinses for a 2-week period. Group 1 used rinse that contained an antiplaque agent, while group 2 received a similar rinse except that it contained no antilpaque agent. A plaque index that measures the plaque buildup was recorded after 2 weeks. The sample mean and sample standard deviation for the 2 groups are shown below:

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>7</td>
</tr>
<tr>
<td>Sample mean</td>
<td>0.78</td>
</tr>
<tr>
<td>Sample standard deviation</td>
<td>0.32</td>
</tr>
</tbody>
</table>

a. State the null and alternative hypotheses that should be used to test the effectiveness of the antiplaque rinse.

b. Assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2$ but unknown, and that the 2 populations follow the normal distribution. Compute the appropriate test statistic.

c. What is your conclusion using $\alpha = 0.05$
Test for the difference between two population means
Sample size determination

Suppose that we want to test the following hypothesis:

\[ H_0 : \mu_1 - \mu_2 = 0 \]
\[ H_a : \mu_1 - \mu_2 > 0 \]

To perform this test a sample of \( n \) observations from each population is to be selected. Assume that the populations are normally distributed with known variances \( \sigma_1^2 \) and \( \sigma_2^2 \) respectively.

a. If we require \( 1 - \beta \) power of the test and we are willing to accept Type I error \( \alpha \), find an expression for the sample size needed to detect a shift in the difference between the two population means from \( \mu_1 - \mu_2 = 0 \) to \( \mu_1 - \mu_2 = \delta \) (\( \delta > 0 \)).

b. Find the sample size needed to detect with probability 90\% a 10 mg/dl difference in cholesterol level between a low-fat diet group and a control group if we can accept a Type I error \( \alpha = 0.05 \). Assume \( \sigma = 28 \text{ mg/dl} \) for both groups.
Power of the test - example:
Let $X$ equal the breaking strength of a steel bar. If the steel bar is manufactured by a certain process, then $X \sim N(50, 6)$. Suppose that a sample of size $n = 16$ will be selected. Find the probability of detecting a shift from $\mu_0 = 50$ to $\mu_a = 55$ if we can accept a Type I error $\alpha = 0.05$. 