Statistics 10
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Some review problems

EXERCISE 1
The lifetime of certain batteries are supposed to have a variance of 150 hours$^2$. Using $1 - \alpha = 0.95$ construct a confidence interval for the population variance $\sigma^2$. It is given that 15 of these batteries (which constitutes a random sample from a normal population) have:

$$\sum_{i=1}^{15} x_i = 250, \quad \sum_{i=1}^{15} x_i^2 = 8000.$$  

where $X$ denotes the lifetime of a battery. What do you conclude?

EXERCISE 2
Suppose that the length in millimeters of metal fibers produced by a certain process follow the normal distribution with mean $\mu$ and standard deviation $\sigma$ (both are unknown). We will test:

$H_0 : \mu = 5.2$
$H_a : \mu \neq 5.2$

A sample size of $n = 15$ metal fibers was selected and was found that $\bar{x} = 5.4$ and $s = 0.4266$.

a. Approximate the p-value using only your $t$ table and use it to test this hypothesis. Assume $\alpha = 0.05$.
b. Assume now that the population standard deviation is known and it is equal to $\sigma = 0.4266$. Compute the power of the test when the actual mean is $\mu_a = 5.35$ and you can accept $\alpha = 0.05$.
c. Draw the two distributions (under $H_0$ and under $H_a$) and show the Type I error and the Type II error on them.
d. Assume now that the hypothesis we are testing is

$H_0 : \mu = 5.2$
$H_a : \mu > 5.2$

Determine the sample size needed in order to detect with probability 95% a shift from $\mu_0 = 5.2$ to $\mu_a = 5.3$ if you are willing to accept a Type I error $\alpha = 0.05$. Assume $\sigma = 0.4266$.

EXERCISE 3
Suppose the nicotine content of two kinds of cigarettes have standard deviations $\sigma_1 = 1.2$ and $\sigma_2 = 1.4$ milligrams. Fifty cigarettes of the first kind had a sample mean content $\bar{x}_1 = 26.1$ milligrams, while 40 cigarettes of the second kind had a sample mean content of $\bar{x}_2 = 23.8$ milligrams. Use $\alpha = 0.05$ to test the following hypothesis:

$H_0 : \mu_1 - \mu_2 = 2$
$H_a : \mu_1 - \mu_2 \neq 2$

EXERCISE 4
A coin is thrown independently 10 times to test that the probability of heads is $\frac{1}{2}$ against the alternative that the probability is not $\frac{1}{2}$. The test rejects $H_0$ if either 0 or 10 heads are observed.

a. What is the significance level $\alpha$ of the test?
b. If in fact the probability of heads is 0.1, what is the power of the test?

EXERCISE 5
On the next plot we see the power against different values of $\mu_a$ for the test:

$H_0 : \mu = 50$
$H_a : \mu > 50$
You are given $\alpha = 0.05, \sigma = 10$. One of the power curves correspond to the case $n = 25$ and the other one to the case $n = 9$. Indicate which power curve corresponds to which sample size with an explanation.

EXERCISE 6
Let $X$ equal the yield of corn in tons per acre per year. Assume that $X$ follows the normal distribution with $\mu = 1.5, \sigma = 0.3$. It is hoped that a new fertilizer will increase the average yield of corn. We will test the hypothesis:

$H_0 : \mu = 1.5$

$H_a : \mu > 1.5$

Assume that the standard deviation continues to equal $\sigma = 0.3$ with the new fertilizer. Using $\bar{x}$, the sample mean of a random sample of size 50 acres, we will reject $H_0$ if $\bar{x} > c$.

a. Find $c$ if we are willing to accept a risk of committing a Type I error $\alpha = 0.05$.

b. Compute the p-value if the sample mean of the 50 observations is $\bar{x} = 1.59$.

c. Suppose a new sample gives $\bar{x} = 1.3$.
   i. Do you reject $H_0$ at the 0.05 level of significance?
   ii. Based on your answer to (i) is it still possible that $H_0$ is false? Explain.