Example 1:

\[ E(N) = E(C \pi r^2) = C \pi Er^2 = C \pi (\sigma^2 + \mu^2). \]

The mean and variance of the distribution of \( r \) are: \( E(r) = 23.5, \text{var}(r) = 1.54. \) Therefore,

\[ E(N) = 8\pi(1.54 + 23.4^2) = 13800.39 \approx 13801. \]

Or simply compute \( E(N) \) as:

\[ E(N) = E(C \pi r^2) = C \pi \sum_r r^2 P(r) = 8\pi \left[ 21^2(0.05) + 22^2(0.20) + \ldots + 26^2(0.05) \right] = 13800.39 \approx 13801. \]

Example 2:

It is given that \( P(X = i) = cP(X = i - 1) \) for \( i = 1, 2. \) Let \( P(X=0)=p. \)

\( P(X = 1) = cP(X = 0) = cp, \) and \( P(X = 2) = cP(X = 1) = c^2p. \) Also, \( P(X = 0) + P(X = 1) + P(X = 2) = 1. \)

Or \( p + cp + c^2p = 1 \Rightarrow p = \frac{1}{1+c+c^2}. \) The expected value of \( X \) is:

\[ E(X) = 0(p) + 1(cp) + 2(c^2p) = \frac{c}{1+c+c^2} + \frac{2c^2}{1+c+c^2} = \frac{c(1+2c)}{1+c+c^2}. \]

Example 3:

There are 8 white, 4 black and 2 orange balls. Two balls are selected without replacement. For each black we win $2, for each white we lose $1.

We neither win nor we lose anything if we select an orange ball.

<table>
<thead>
<tr>
<th>Color</th>
<th>X($)</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WW</td>
<td>-2</td>
<td>1/5</td>
</tr>
<tr>
<td>WO or OW</td>
<td>-1</td>
<td>2/5</td>
</tr>
<tr>
<td>OO</td>
<td>1</td>
<td>2/5</td>
</tr>
<tr>
<td>BB</td>
<td>2</td>
<td>2/5</td>
</tr>
<tr>
<td>BO or OB</td>
<td>4</td>
<td>1/5</td>
</tr>
</tbody>
</table>

b. Expected value of our winnings:

\[ E(X) = \sum_{i=-2}^{2} X_i \cdot P(X_i) = -2 \cdot \frac{1}{5} + (-1) \cdot \frac{2}{5} + 0 + \frac{2}{5} + 1 \cdot \frac{2}{5} + 2 \cdot \frac{2}{5} + 4 \cdot \frac{1}{5} = 0. \]

c. Standard deviation of our winnings:

\[ \sigma^2 = \sum_{i=-2}^{2} (X_i - E(X))^2 \cdot P(X_i) = 2^2 \cdot \frac{1}{5} + 1^2 \cdot \frac{2}{5} + 0^2 \cdot \frac{2}{5} + 1^2 \cdot \frac{2}{5} + 2^2 \cdot \frac{2}{5} + 4^2 \cdot \frac{1}{5} = \frac{576}{125}. \]

Therefore the standard deviation is \( \sigma = \sqrt{\frac{576}{125}} = 1.78. \)

d. \( P(X = -2 | X < 0) = \frac{P(X = -2 \cap X < 0)}{P(X < 0)} = \frac{P(X < 0)P(X = -2)}{P(X < 0)} = \frac{1 - \frac{1}{1+c+c^2}}{\frac{1}{1+c+c^2}} = \frac{56}{88} = 0.64. \)

Example 4:

Let \( X \) be the number of tests needed for each group of 10 people. Then, if nobody has the disease 1 test is enough. But if the test is positive then there will be 11 test (1 + 10). The probability distribution of \( X \) is:

\[ X \quad P(X) \]
\[ 1 \quad \frac{10!}{0!} \cdot 0.99^{10} \cdot 0.01^{0} = 0.99^{10} \]
\[ 11 \quad 1 - \frac{10!}{0!} \cdot 0.99^{10} \cdot 0.01^{0} = 1 - 0.99^{10} \]

Therefore the expected number of tests is:

\[ E(X) = 1 \cdot (0.99)^{10} + 11 \cdot (1 - 0.99^{10}) = 7.51. \]

Example 5:

Using example 4 when \( n = 2, 4, 5, 20 \) we get the following: When \( n = 2, \) \( E(X) = 1.38. \) The total number of tests for the 100 people is 1.38(50) = 69.

When \( n = 4, \) \( E(X) = 2.38. \) The total number of tests for the 100 people is 2.38(25) = 59.5.

When \( n = 5, \) \( E(X) = 3.05. \) The total number of tests for the 100 people is 3.05(20) = 61.

When \( n = 20, \) \( E(X) = 18.57. \) The total number of tests for the 100 people is 18.57(5) = 92.9.

Therefore to minimize the number of tests we must place them in groups of 4.

Example 6:

The horse will win both races with probability 0.06, one race with probability 0.38, and no race with probability 0.56. The probability distribution of the profit \( X \) will be:

\[ X \quad P(X) \]
\[ 80000 \quad 0.06 \]
\[ 30000 \quad 0.38 \]
\[ -10000 \quad 0.56 \]

The expected value and standard deviation of the profit are:

\[ E(X) = 80000(0.06) + 30000(0.38) - 10000(0.56) = 10600. \]

\[ SD(X) = \sqrt{80000^2(0.06) + 30000^2(0.38) + 10000^2(0.56) - 10600^2} = 25877.40. \]