EXERCISE 1
A coin is thrown independently 10 times to test that the probability of heads is \( \frac{1}{2} \) against the alternative that the probability is not \( \frac{1}{2} \). The test rejects \( H_0 \) if either 0 or 10 heads are observed.

a. What is the significance level \( \alpha \) of the test?

b. If in fact the probability of heads is 0.1, what is the power of the test?

EXERCISE 2
Suppose that \( X \sim \text{bin}(100, p) \). Consider the test that rejects \( H_0 : p = 0.5 \) in favor of \( H_a : p \neq 0.5 \) for \(|X - 50| > 10\). Use the normal approximation to binomial distribution to answer the following:

a. What is \( \alpha \)?

b. Graph the power as a function of \( p \).

EXERCISE 3
True or false:

a. If the sample size is decreased when testing a hypothesis, the power would be expected to increase.

b. If the significance level of a test is decreased, the power would be expected to increase.

c. If a test is rejected at the significance level \( \alpha \), the probability that the null hypothesis is true equals \( \alpha \).

d. The probability that the null hypothesis is falsely rejected is equal to the power of the test.

e. A Type I error occurs when the test statistic falls in the rejection region of the test.

f. In testing a hypothesis, when the difference between the hypothesized mean and the actual mean (shift from \( \mu_0 \) to \( \mu_a \)) is increased, the power of the test will decrease.

g. In testing a hypothesis, when the standard deviation is decreased, the power of the test will decrease.

EXERCISE 4
Let \( X_1, \cdots, X_{25} \) be a random sample from a normal distribution having a variance of 100. Find the rejection region for a test at level \( \alpha = 0.10 \) of \( H_0 : \mu = 0 \) against \( H_a : \mu > 0 \). What is the power of the test when the actual mean is \( \mu = 1.5 \)? Repeat for \( \alpha = 0.01 \).

EXERCISE 5
Advertisements claim that the average nicotine content of a certain kind of cigarette is only 0.30 milligram. Assume that the population standard deviation is 0.15 milligram. Suspecting that this figure is too low, the health department of the City of Los Angeles and the consumer protection service take a random sample of 121 of those cigarettes from different production lots. They find that the sample mean is \( \bar{x} = 0.33 \) milligram.

a. Use the 0.05 level of significance to test if the nicotine content is more than 0.30 milligram.

b. If the actual population mean nicotine content is 0.31 milligram, compute the type II error \( \beta \), and the power of the test \( 1 - \beta \).

EXERCISE 6
Investigating the possibility that a coin-operated soda vending machine is dispensing too much soda, the owner of the machine takes a random sample of 41 “6-fluid-ounce” servings and finds that the sample mean is \( \bar{x} = 6.15 \) fluid ounces with sample standard deviation \( s = 0.3 \) fluid ounces.

a. Is there evidence of overfilling? Use 0.01 level of significance.

b. Explain to a friend of yours who has not taken statistics yet, what would be a type I error and a type II error in testing the hypothesis of part (a).
EXERCISE 7
A manufacturer claims that 20% of the public preferred her product. A sample of 100 persons is taken to check her claim. It is found that 8 of these 100 persons preferred her product.

a. Using the 0.05 level of significance test her claim (perform a two-sided test).

b. Find the p-value and explain what it means.

EXERCISE 8
Last month, a large supermarket chain received many consumer complaints about the quantity of chips in 16-ounce bags of a particular brand of potato chips. Suspecting that the complaints were merely the result of the potato chips settling to the bottom of the bags during shipping, but wanting to be able to assure its customers they were getting their money’s worth, the chain decided to test the following hypotheses concerning the mean weight (in ounces) of a bag of potato chips in the next shipment of potato chips received from their largest supplier:

$H_0 : \mu = 16$

$H_a : \mu < 16$

If there is evidence that $\mu < 16$, then the shipment would be refused and a complaint registered with the supplier.

a. What is a Type I error, in terms of the problem?

b. What is a Type II error, in terms of the problem?

c. Which type of error would the chain’s customers view as more serious? Which type of error would the chain’s supplier view as more serious?

EXERCISE 9
The output voltage of a certain electric circuit is specified to be 130 volts. The population standard deviation is known to be $\sigma = 3.0$ volts. A sample of 40 readings on the voltage of this circuit gave a sample mean of 128.6 volts.

a. Test the hypothesis that the mean output voltage is 130 volts against the alternative that it is less than 130 volts. Use $\alpha = 0.05$.

b. Based on your answer to (a), is it possible that the mean output voltage is still 130 volts? Explain.

c. If the true population mean output voltage is 128.6 volts, compute the probability of a type II error ($\beta$) and the power of the test ($1 - \beta$) when $\alpha = 0.05$.

d. For this part you do not have to show any calculations.
   How would the type II error $\beta$ be affected if:
   i. The type I error $\alpha$ decreases to 0.01?
   ii. The true population mean is 129.6 volts?

EXERCISE 10
Suppose that we wish to test the null hypothesis, $H_0$, that the proportion of ledger sheets with errors, $p$, is equal to 0.05 against the alternative, $H_a$, that the proportion is larger that 0.05 by using the following scheme. Two ledger sheets are selected at random. If both are error free, we reject $H_0$. If one or more contains an error, we look at a third sheet. If the third sheet is error free, we reject $H_0$. In all other cases we accept $H_0$.

a. What is the value of $\alpha$ associated with this test?

b. Calculate the type II error $\beta$ as a function of $p$. 