Constructing the optimal portfolios - Constant correlation model
Calculation steps

a. Step 1: Compute the historical mean return, standard deviation for each stock. You will also need the correlation coefficients for all pairs of stocks (step 2). Construct the table below:

<table>
<thead>
<tr>
<th>Stock</th>
<th>( \bar{R}_i )</th>
<th>( R_i - R_f )</th>
<th>( \sigma_i )</th>
<th>( \frac{R_i - R_f}{\sigma_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOOGLE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Step 2: Sort the table above based on the excess return to standard deviation ratio:

\[
\frac{\bar{R}_i - R_f}{\sigma_i}
\]

c. Step 3: Create 3 columns to the right of the sorted table as follows:

<table>
<thead>
<tr>
<th>Stock</th>
<th>( \bar{R}_i )</th>
<th>( R_i - R_f )</th>
<th>( \sigma_i )</th>
<th>( \frac{R_i - R_f}{\sigma_i} )</th>
<th>( \rho )</th>
<th>( 1 - \rho + i \rho )</th>
<th>( \sum_{j=1}^{i} \frac{R_j - R_f}{\sigma_j} )</th>
<th>( C_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: \( \rho \) is the average correlation. It is equal to:

\[
\rho = \frac{\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \rho_{ij}}{n(n-1)}
\]

Note: Compute all the \( C_i \), \( i = 1, \ldots, n \) (last column) as follows:

\[
C_i = \frac{\rho}{1 - \rho + i \rho} \sum_{j=1}^{i} \frac{\bar{R}_j - R_f}{\sigma_j} = COL1 \times COL2.
\]

Once the \( C_i \)'s are calculated we find the \( C^* \) as follows:

If short sales are allowed, \( C^* \) is the last element in the last column.
If short sales are not allowed, \( C^* \) is the element in the last column for which \( \frac{\bar{R}_i - R_f}{\sigma_i} > C^* \).

In both cases the \( z_i \)'s are computed as follows

\[
z_i = \frac{1}{(1 - \rho) \sigma_i} \left[ \frac{\bar{R}_i - R_f}{\sigma_i} - C^* \right]
\]

and the \( x_i \)'s

\[
x_i = \frac{z_i}{\sum_{i=1}^{n} z_i}
\]