The single index model states that
\[ R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \]
where, \( R_{it} \) is the return of stock \( i \) at time \( t \) and \( R_{mt} \) is the return of the market at time \( t \).

Assumptions and notation:
\[ E(\epsilon_i) = 0, \quad var(\epsilon_i) = \sigma^2_{\epsilon_i}, \quad E(\epsilon_i \epsilon_j) = 0, \quad var(R_m) = \sigma^2_m, \quad E(R_m) = \bar{R}_m. \]

Therefore,
\[ E(R_i) = \alpha_i + \beta_i \bar{R}_m \]
\[ var(R_i) = \sigma^2_i = \beta_i^2 \sigma^2_m + \sigma^2_{\epsilon_i} \]
\[ cov(R_i, R_j) = \sigma_{ij} = \beta_i \beta_j \sigma^2_m \]

Here are some useful formulas:

a. Estimate of \( \beta_i \) (beta of stock \( i \)):
\[ \hat{\beta}_i = \frac{\sum_{t=1}^m (R_{it} - \hat{\alpha}_i - \hat{\beta}_i R_{mt})}{\sum_{t=1}^m (R_{mt} - \bar{R}_m)^2}. \]

b. Estimate of \( \alpha_i \) (alpha of stock \( i \)):
\[ \hat{\alpha}_i = \bar{R}_i - \hat{\beta}_i \bar{R}_m. \]

c. Estimate of \( \sigma^2_{\epsilon_i} \) (variance of random error term associated with stock \( i \)):
\[ \hat{\sigma}^2_{\epsilon_i} = \frac{\sum_{t=1}^m \epsilon^2_{it}}{m - 2} = \frac{\sum_{t=1}^m (R_{it} - \hat{\alpha}_i - \hat{\beta}_i R_{mt})^2}{m - 2}. \]

d. Estimate of \( var(\hat{\beta}_i) \).
\[ var(\hat{\beta}_i) = \frac{\hat{\sigma}^2_{\epsilon_i}}{\sum_{t=1}^m (R_{mt} - \bar{R}_m)^2}. \]

e. Correlation between stock \( i \) and stock \( j \):
\[ \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \frac{\beta_i \beta_j \sigma^2_m}{\sigma_i \sigma_j}. \]

f. Correlation between stock \( i \) and market:
\[ \rho_{im} = \beta_i \frac{\sigma_m}{\sigma_i} \Rightarrow \beta_i = \rho_{im} \frac{\sigma_i}{\sigma_m}. \]
Simple R commands:

```r
a1 <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c183_c283/stocks5_period1.txt", header=TRUE)

#Regression of r11 on rsp1 (index):
q <- lm(a1$r11 ~ a1$rsp1)

#Summary of the regression above:
summary(q)

#List the names of the results in object q:
names(q)

#Get the estimates of alpha and beta:
q$coefficients[1]
q$coefficients[2]

#List the residuals:
q$residuals

#Get the estimate of the variance of the error term (MSE):
sum(q$residuals^2)/(nrow(a1)-2)

#Another way:
summary(q)$sigma^2

#variance-covariance matrix of the estimates of the main parameters
#of the model:
vcov(q)

#Get the variance of the estimate of beta:
vcov(q)[2,2]

#Another way:
summary(q)$coefficients[4]^2
```